

BROWN AND SMITH

Mr. Brown one day noticed that if his age was written backwards it gave the age of his friend Mr. Smith. (For example, Mr. Brown might have been 32 and Mr. Smith 23. But do not think that their ages actually were these numbers ; this is only an example of what they might have been.)

Some questions occurred to Mr. Brown. (1) Will it still be true in a year's time that Mr. Smith's age is Mr. Brown's written backwards ? (2) Will it always be true ? (3) Will it ever be true again? If so, when ? (4) If any two friends wait long enough, will the time come when the age of one of them is the age of the other written backwards ?

SOLUTIONS.

BROWN AND SMITH.

(1) It will certainly not be true next year, because the numbers in the ten's column cannot keep up with the numbers in the unit column. For instance, if Mr. Brown was 38 and Mr. Smith 83, next year Mr. Smith would be 84, and Mr. Brown would have to be this backwards, that is, 48. But Mr. Brown cannot leap from 38 to 48 in a single year. (2) So it certainly cannot always be true. (3) But it should happen every eleven years. If Mr. Brown is 38 and Mr. Smith 83, in eleven years they will be 49 and 94, and eleven years ago they were 27 and 72. There is trouble, of course, if Mr. Smith gets to be more than a hundred, or if we go back to the time before Mr. Brown was born! (4) No, as a rule it is not possible for two friends to wait until this happens. The difference between 32 and 23 is 9, between 38 and 83 is 45 which is a multiple of 9. If one age is the other written backwards, you will always find the difference is a multiple of 9. So, unless the difference between the ages of the friends is one of the numbers 9, 18, 27 ... ~~it will never happen.~~ it will never happen. Of course, they could be the same age, since 0 counts as a multiple of 9. Then it would happen, in a rather special way, when they were ~~11, 22, 33, 44, 55, 66, 77, 88, 99~~ 11 years old, 22 years old, 33 years old, and so on up to 99.

PUZZLES AND GAMES.

Spring

THE LIFE OF EASE.

"You know," said George, "Life is not really so bad. We only work eight hours a day. That's only a third of our life. In ~~the year~~, 1956 for instance, which is leap year with 366 days, that means we put in 122 days work. But we don't work on Saturdays and Sundays. As there are 52 Saturdays and 52 Sundays in a year, that means you can knock off 104 days, and 104 from 122 leaves only 18. Then we have a fortnight's holiday at Christmas, another 14 days to knock off, which only leaves 4 days. And there are at least 4 public holidays in the year. That means, we don't do any work at all. "

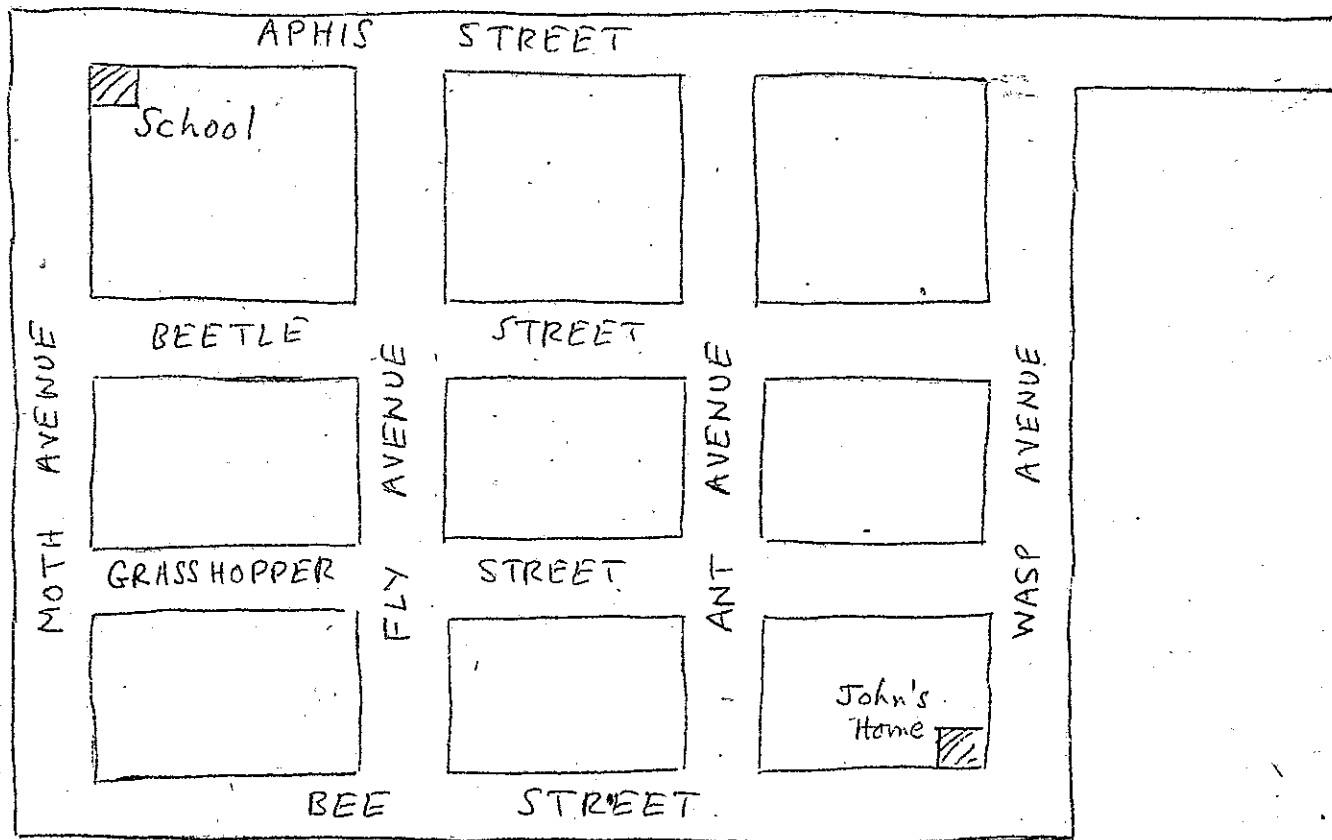
Now there surely must be something wrong in George's argument. But just what is it ?

TIGERS AND ELEPHANTS.

This is a game played by children in India. First, you draw the shape shown here, a pentacle. One player has a single black counter ; this is the tiger. The other player has six white counters - the elephants. First of all the tiger is put on the board, in any one of the ten positions marked. Then one of the elephants is put on. Now the tiger moves. A move is from ~~any dot~~ where the tiger is, along a line to the next dot, provided this dot is empty. Suppose for instance, the tiger started on 6 and the first elephant was put down on 9. Then the tiger can move to 2, or 3, or 8, whichever it likes. When it has moved, a second elephant is put on, and the tiger moves again. So it continues until all the elephants are on the board. they too are allowed

BUGWORTHY.

This is the plan of the township of Bugworthy. Whoever laid it out had a tidy mind, but not much imagination.



John Buggina lives at the corner of Bee Street and Wasp Avenue. Every day he cycles to school. To avoid boredom, he tries to cycle to school a different way each day, but of course not going further than he has to. Each day he returns by the route he used that morning. For how many days can he cycle to school without repeating the same route ?

A JIGSAW PUZZLE.

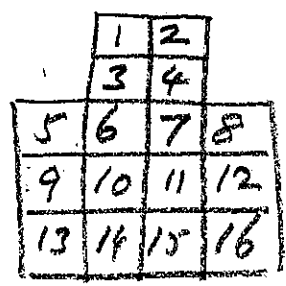
There are some puzzles that are perfectly fair, but when you are told the answer you say, "Oh, I never thought of that." I thought it would be interesting to give you a puzzle of this kind, then tell you the answer, and then see if you could do a second puzzle based on exactly the same idea. I suggest you stop reading when you come to the line of stars below, and have a good try at the first puzzle. If you have not solved it after trying it for a week or two, read on below the stars.

Here is the first puzzle. (INSERT DIAGRAM I.) Can you cut this diagram of sixteen squares into two pieces, which will fit together and make a perfect square? You do not need to cut across any of the little squares, and I will give you a hint; each piece contains eight squares. Now stop reading!

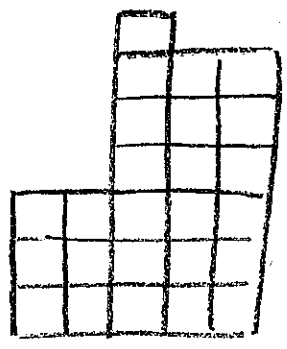
Solution to the first puzzle. One piece contains the squares marked 1, 2, 3, 4, 6, 7, 8, 10. If you cut this piece out, and turn it so that squares 10, 6, 8, 1 are at the top, and square 8 sticks out at the bottom, you will easily fit it into the other piece to make a big square.

The second puzzle. (INSERT DIAGRAM II.) Here is a diagram of twenty-five squares. Can you cut it into two pieces that will join together to make a perfect square?

I



II



PUZZLES AND GAMES

How to use these puzzles.

There is a riddle that asks, "Why was Saul surprised when David's stone killed him ? " I do not think anybody could guess the answer , "Because such a thing had never entered his head before." Some of these puzzles are like that ; the answer is a clever trick that nobody is expected to work out, but you are surprised or amused when you look at the solution and see what it is.

Other puzzles are ones that, if you keep trying them long enough - say for a week, longer if you are a very determined person - you ought to find the answer, or a fair part of the answer.

Still other puzzles are what I call "Investigations". With these, you have to work away until you have a lot of information, and then you have to look at this information and see if it suggests anything to you. You can take a month over this, if you like.

The games are, first of all, games to play ; sometimes, but not always, these games have a secret. If you can find out the secret, you can be sure of winning every time.

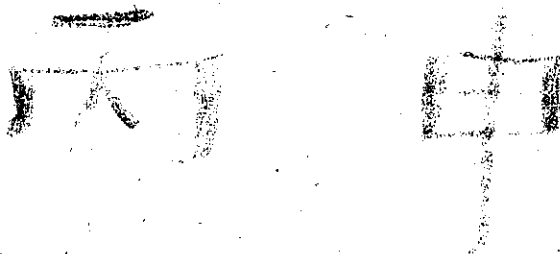
The Four Fours.

Many numbers can be written by using the figure 4 exactly ~~the~~ four times, with as many + , - , x , / signs as you like. For example, you can write 12 as $\frac{44 + 4}{4}$, or 15 as $44 + \frac{4}{4}$, or 64 as $(4+4) \times (4+4)$. The brackets, in this last one, mean that you do the additions before the multiplication, so that you get 8 x 8.

every number

The puzzle is, can you write ~~all the numbers~~ from 1 to 10 inclusive in this way ?

No special trick in this ; if you and your friends keep pegging away at it you should find the answers in time on most of them.



Clarifier for
all letters
& numbers
of years.

The signs above show how the Chinese will write the year we call 1956. I think the best translation I can give is to say that these signs mean C 9 . Every Chinese year has a "letter" and a "number." There are ten "letters" which I will denote by A, B, C, D, E, F, G, H, I, J ; the Chinese call them "Heaven's Stems" and use much more beautiful signs for them. There are twelve "numbers", for which I use our ordinary numbers 1 to 12. (The Chinese name is "Earth's Branches".) Every year both the "letter" and the "number" change. Thus, 1924 A.D. was an A 1 year ; the next year, 1925, was B 2 ; the year after, C 3 , and so on. 1933 was J 10. After J, A begins again , so 1934 would be A 11, and 1935 B 12. After 12, we start again with 1. So 1936 is C 1. And so it goes on. Eventually ~~XXXXXXXXXX~~ the signs begin to repeat themselves ; we come again to an A 1 year. From ~~XXXXXXXXXXXX~~ one A 1 year to the next A 1 year is called a cycle. The question is, how long is a cycle ? How long have we to wait until the next A 1 year comes round ?

This question is a rather easy "Investigation". If the worst comes to the worst, you can write down all the years of a Chinese cycle, and see how many there are. But you can reduce the labour considerably if you hit on a suitable trick.

A GAME WITH STONES.

This is a game for two players. The first player puts on the ground a fair number of stones - round about fifteen, say. The second player then removes either one or three stones, whichever he likes. Then the first player removes either one or three. And so it goes on, the players taking stones alternately. It does not matter what you have done in past moves; at each move you may take one or three stones, whichever you prefer (but not two!). The player who takes the last stone from the ground has lost.

Play this game for a ~~month~~^{while} or two, and see if you can find the secret of ~~always winning it~~, which is a rather surprising one in some ways.

SOLUTIONS.

THE FOUR FOURS.

There are various solutions. One way is as follows -

1 = 44/44 ; 2 = 4/4 + 4/4 ; 3 = (4+4+4)/4 ; 4 = 4 + 4x(4 - 4)

5 = (4 x 4) + 4/4 ; 6 = (4 + 4) + 4 ; 7 = 4 + 4 - 4/4

8 = 4 x (4 + 4)/4 ; 9 = 4 + 4 + 4/4 ; 10 = (44 - 4)/4

CHINESE CALENDAR.

Since there are 10 letters, A will turn up every 10 sign years ; that is to say, there will be an A in the year 10, 20, 30, 40, ... years after the beginning of a cycle. Since there are 12 numbers, the number 1 will turn up every 12 years ; that is to say, there will be a 1 in the year 12, 24, 36, 48, 60 years after the beginning of a cycle. You might picture it like this:

A.....A.....A.....A.....A.....A.....A
1.....1.....1.....1.....1.....1.....1

You can see that after 60 years from an A 1 year, the next A 1 year will come, and that this is the first time that an A and a 1 come together again. 60 in fact is the smallest number that divides both by 10 and by 12. So there are 60 years in a cycle, and the next A 1 year will be 60 years after 1924, that is, in 1984.

A GAME WITH STONES.

If the first player places an odd number of stones on the ground, he is sure to win. It does not matter how he plays during the game itself ! The reason is this. 1 and 3 are both odd numbers. If you take an odd number away from an odd number, the result is even. If you

PQ12

take an odd number away from an odd number, the result is even. If you take an odd number away from an even number, the result is odd. Now at every move of each player, an odd number of stones are taken from the pile.

~~XXXXXXXXXXXXXXXXXXXX~~

So, every time the second player has just moved there will be an even number of stones remaining, and every time the first player has just moved, there will be an odd number left. (Try this out, and check that it is so; remember the first player puts down an odd number of stones, and the other player takes first.)

A player loses when he takes the last stone, that is when he leaves no stone on the ground. Nought is an even number; as the first player always leaves an odd number, he cannot possibly leave nought ~~XXXXXX~~. So the first player cannot lose. So the second player must lose.

THE SEPTEMS

In a little known island, roughly in latitude 97° S, there live a curious race known as the Septems. A Septem has only one hand, and this hand has seven fingers. When a Septem counts, he presses his fingers in turn on his nose.

Just as our counting is based on ten, because we have ten fingers, so the Septems' counting is based on seven. When we write 23 we mean 2 tens and 3 units. When a Septem writes 23 he means 2 sevens and 3, and he reads it "twentug-three". "Tug" is the Septem name for what we call "seven". Fortunately the Septems press very heavily when they write, so you can always tell Septem numbers by their being so black. Their 30 (thirtug) is our 21; 40 (fortug) is our 28. The only numbers they use, of course, are 0, 1, 2, 3, 4, 5, 6. Our 7 is written 10 (tug); then comes 11 (tug-one, our 8) and 12 (tug-two, our 9).

Just as we have the word "hundred" for ten tens, they have the word "plonk" for seven sevens. Instead of hundreds, tens and units columns, there are plonks, tugs and units. 144 means 1 plonk, 4 tugs and 4 units; that is, $49 + 28 + 4 = 81$, in our numbers. Try counting from "one" to "plonk" in Septem language.

Can you write out the Septem addition table, from 0+0 to 6+6? And the multiplication table from 1 x 1 to 12 x 12? (Remember 12 is not "twelve" but "tug-two" or "nine"!)

In our arithmetic you can test whether a number divides by 9 by adding up the digits., e.g. 72 divides by 9, since $7+2 = 9$. Is there any table in the Septem language arithmetic where the same kind of thing happens?

SOLUTIONS.

THE SEPTEMS.

Counting. The best thing is to get a friend to check you from ~~this~~ the answer below, while you count from "one" to "plonk".

One, two, three, four, five, six, tug. Tug-one, tug-two, tug-three, tug-four, tug-five, tug-six, twentug. Twentug-one, twentug-two, twentug-three, twentug-four, twentug-five, twentug-six, thirtug. Thirtug-one, thirtug-two, thirtug-three, thirtug-four, thirtug-five, thirtug-six, fortug. Fortug-one, fortug-two, fortug-three, fortug-four, fortug-five, fortug-six, fiftug. Fiftug-one, fiftug-two, fiftug-three, fiftug-four, fiftug-five, fiftug-six, sixtug. Sixtug-one, sixtug-two, sixtug-three, sixtug-four, sixtug-five, sixtug-six, plonk.

Addition table.

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	10
2	2	3	4	5	6	10	11
3	3	4	5	6	10	11	12
4	4	5	6	10	11	12	13
5	5	6	10	11	12	13	14
6	6	10	11	12	13	14	15

All figures black type in...

Multiplication table.

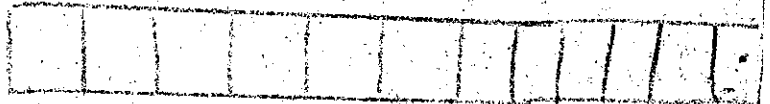
	1	2	3	4	5	6	10	11	12
1	1	2	3	4	5	6	10	11	12
2	2	4	6	11	13	15	20	22	24
3	3	6	12	15	21	24	30	33	36
4	4	11	15	22	26	33	40	44	48
5	5	13	21	26	34	42	50	55	63
6	6	15	24	33	42	51	60	66	105
10	10	20	30	40	50	60	100	110	120
11	11	22	33	44	55	66	110	121	132
12	12	24	36	51	63	105	120	132	144

The 6 times table contains the numbers 6, 15, 24, 33, 42, 51, 60, 66, 105 all of which add up to a multiple of 6.

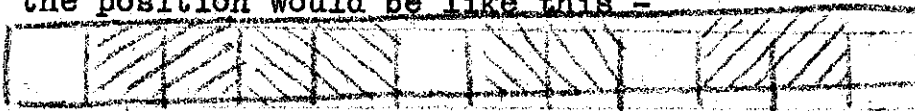
It is interesting to notice that some results of ours are still

THE GAME OF OBLONG

Twelve squares are drawn, as in the diagram. Two players play



in turn. Each move consists in blacking out two squares, which must be next to each other. When a player cannot find two neighbouring squares to black out, the game ends and that player has lost. For instance, if player A at his first move blacked out squares 2 and 3, then B did 7 and 8, then A 10 and 11, then B 4 and 5, the position would be like this -



Now A cannot find two neighbouring squares, so A has lost.

Who should win, the first player or the second? And how is it to be done?

There is a very simple rule for winning, but it is not at all easy to guess.

SOLUTIONS.

THE GAME OF OBLONG.

A, the player who plays first, should win. A's first move should be to blacken squares 6 and 7, the middle ones. Then, whatever B does on one side of the board, A does the same thing on the other side. If B does 1 and 2, then A does 12 and 11 at his next turn. If B then does 8 and 9, A does 5 and 4.

By this rule, A is bound to win. For after every move of A's there is a balanced pattern on the board ; if B can find a space to get into on the right, there must be another space just like it for A to use on the left. If B can find a space on the left, A can find an exactly similar space on the right.

to move. Only one elephant is allowed to move at a time, of course, and the two players take it in turn to move. The elephants move in the way described above - from a dot, along a line, to a neighbouring dot.

There is one thing the tiger is allowed to do, that the elephant cannot do. A tiger can jump over an elephant, provided there is an empty space on the other side of the elephant for the tiger to land in. This is rather like in draughts. If the tiger can catch only one elephant in this way, the tiger has won. Remember that the elephants cannot move until all six are on the board; the elephant player must be very careful where he puts them! When the tiger has jumped over an elephant, that elephant is removed from the board.

How do the elephants win? They win, if they can force the tiger into a position where it cannot move at all.

Please note that this game is not dealt with in the solutions. I only heard about it recently from an Indian friend, and I have not quite finished working out the theory of it. My impression is that the tiger should not win unless the elephants are very careless indeed. But I do not think the elephants can be sure of winning; in actual games they often do win, because the tiger makes a mistake. Try playing the game, and see if you agree with this opinion.

SOLUTIONS.

THE LIFE OF EASE .

George is quite right at the beginning, when he says that working 8 hours a day for 366 days is the same total as 122 days of 24 hours. But then not working on Saturday means the loss of only 8 hours work, which George regards as one-third of a day. So, when he knocks off 52 Saturdays, * he should subtract only 52 thirds of a day. The same applies to all the other holidays he mentions.

BUGWORTHY.

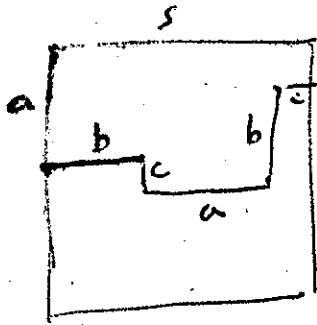
To get to school, John has to go three blocks to the West and three to the North. The choice he has is in which order to do these. For instance, if he decided to go all the way along Bee Street and then all the way up Moth Avenue, this would mean he went West-West-West-North-North-North. If he did West-North-West-North-West-North this would give him a route shaped like a staircase. Whichever way he goes there must be three "West"s and three "North"s. We could write the two ways just mentioned, for short, as WWWNNN and WNWNWN. The only difficulty is to make sure that we do not leave any possible ways out, when we write out our list. One way of making sure is the following. In WWWNNN, the Ws come in the first, second and third place, so we could label this 123. The Ns of course fill the remaining three spaces. The label for WNWNWN will be 135, to show that W is found in the 1st, 3rd and 5th places. So the puzzle is really the same as if we had two teams, West and North, each with three members, running in a race, and we had to write out all the different ways in which the West team could score places. It is not hard to write this out in an orderly manner, as is done below.

This gives first the label (like 123) and then the route (WWWNNN) corresponding to this label.

LABEL	ROUTE	LABEL	ROUTE	LABEL	ROUTE
123	WWWNNN	234	NWWWNN	456	NNNWWW
124	WWNWNW	235	NWWNWN	=====	
125	WWNNWN	236	NWWNNW		
126	WWNNNW				
-----		245	NWNWWN		
134	WNWWWN	246	NWNWNW		
135	WNWNWN	-----			
136	WNWNNW	256	NWNNWW		
-----		-----			
145	WNNWWN	345	NNWWWN		
146	WNNWNW	346	NNWNNW		
-----		-----			
156	WNNNWW	356	NNWNWW		
=====		=====			

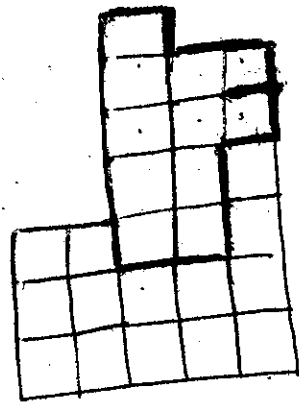
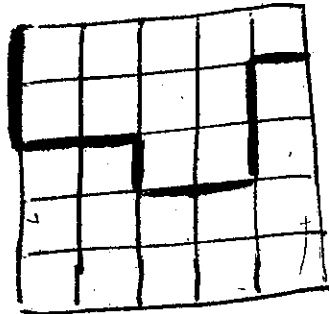
This table shows that there are just 20 different routes by which John could go to school, without going unnecessarily far. So he could cycle on 20 school days without repeating his route to school.

- Winter 1954. 7 Dissection of Swastika ; how many squares . Cat & Mouse
- Spring 1954. Long Division, Additions in Code ; Game with Stones ;
Fifteen Puzzle.
- Summer 1954. Parking. Shunting. Scribbles.
- Autumn 1954. Morse Code. Bowling Averages. Jugs.
- Winter 1955. Game, Oblong. Brown & Smith. Septems.
- Spring 1955. Life of Ease. Tigers & Elephants. Bugworthy.
- Summer 1955. Indian Draughts. Number Squares. St. Skinnians.
- Autumn 1956. Introduction to new year. Four 4's. Chinese calendar. Stones (1 and 3)
- Winter 1956. Word changing. Dots & squares
- Spring 1956. Jigsaw puzzle : magic apples : change for 10/- : Stones (1 and 4)
- Summer 1956. Sums in code
- Prisoners release Sums in code



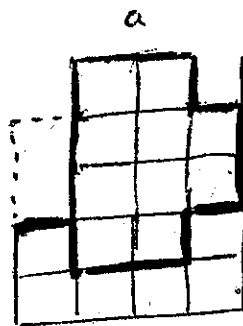
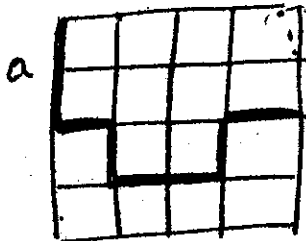
$a + b + c = s$ otherwise as you like
except that $a + c > b$.

$a = 2 \quad b = 2 \quad c = 1 \quad s = 5$



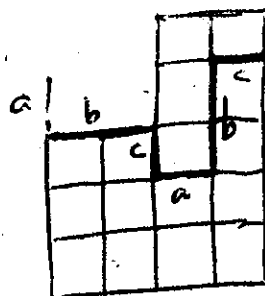
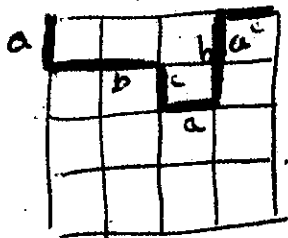
11/14

$a = 2 \quad b = 1, c = 1$

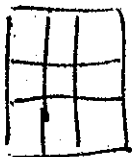


There is simpler solution
when $a + c - b = a$
i.e. when $b = c$.

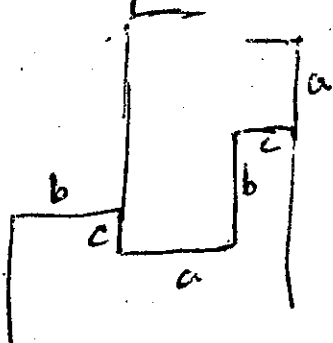
$a = 1, b = 2, c = 1$



Here again, a simpler
solution.



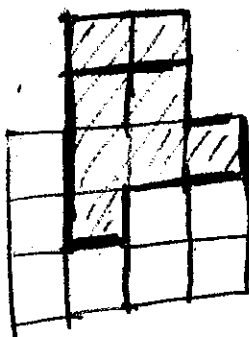
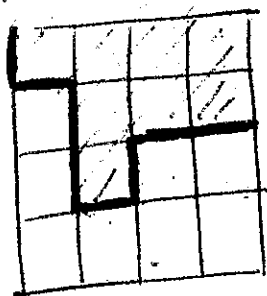
Flat top occurs when $a+c=b$



We must have $a+c > b$

If $s=4$, $b < 2 \therefore b=1$.

$b \neq c$ $c=2$ $a=1$



Each contains 8 squares.

Change for 10/-

Dissection.

The stars same 1 & 4.

Applewoman.

Have a dissection like 15 for next issue.

THE MAGIC APPLES.

This is an old puzzle. It is like a riddle ; no one ever guesses it, but it shows that something that seems impossible actually can be done.

Seven applewomen went to market. The first had 20 apples, the second 40, the third 60, the fourth 80, the fifth 100, the sixth 120, the seventh 140 apples. Each woman carried a notice, saying on what terms she would sell her apples. All the notices were exactly the same, and the women kept strictly to what the notices said. All the apples were sold, and each woman received exactly the same amount of money. How was this possible ? (The apples were not given away for nothing,) but of course there was something strange about the business methods.)

CHANGE FOR TEN SHILLINGS.

I have only silver ~~xxxxx~~ coins in my pocket. I am unable to change a ten shilling note for a friend. What is the largest amount of ~~xxxxx~~ money I can have in my pocket ? (The standard silver coins are half-crown, florin, shilling, sixpence and threepenny bit.)

ANOTHER GAME WITH STONES.

In ~~xxxxx~~ the Autumn 1956 issue of this Journal, I described a game with stones, which there was a very simple secret for winning. There are many games with stones (another one was described in Spring 1954) and quite a little alteration in the rules makes the play entirely different. Can you find out how to win with the following rules ? The first player puts down a heap of stones (more than twenty, say). The players then pick up the stones in turn. You are allowed to pick up four stones or one stone, whichever you prefer, when it is your turn. (At each turn you have a free choice, 4 or 1 ; it does not matter what you did in earlier turns.) ~~xxxxxxxxxx~~
~~xxxxx~~ The first player decides how many stones to put down (any number over 20) ; the second player then picks up, and after that the

two players continue picking up alternately. The player who picks up the last stone has lost. If you ~~were~~ are first player, how many stones should you put down, and how should you play to win? Try playing this game with your ~~five~~ friends. I warn you - the secret is quite different from the rule in the Autumn issue.

SOLUTIONS.

A JIGSAW PUZZLE.

The shaded squares ~~xx~~ form one piece, the unshaded squares the other. If you want to make up puzzles like this for yourself, the best way is to start with the square, and see what fancy shapes you can get from it. These puzzles are much easier to make up than to solve. Make one up and try it on your friends.

(INSERT DIAGRAM III.)

SOLUTIONS.

THE MAGIC APPLES:

The notices said, "Seven apples a penny, but if you buy less than seven you must pay 3d. each for them." So the woman with 140 apples sold all hers in sets of seven, and got twenty pennies. The woman with 20 apples sold two sets of seven, and received two pennies for these, but the remaining six apples had to be sold at 3d. each, thus bringing in eighteen pennies. So she had twenty pennies in all. If you work out what happened to the others, you will find that each of them had twenty pennies too.

CHANGE FOR TEN SHILLINGS.

The largest amount I can have is $15/9$, made up of three half-crowns, four florins and one threepenny bit. With these coins it is impossible to make up ten shillings exactly.

ANOTHER GAME WITH STONES.

If you are first player, you should put down 21 stones. Then, whatever your opponent does, you will find that you can leave him the numbers 16, 11, 6, 1. The reason is that these numbers go down by fives. If he takes 1, you take 4; if he takes 4, you take 1. Either way, the result of his turn and your turn together is to remove 5 stones. So he has no control over what happens.